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ABSTRACT

Geometer's Sketchpad is an interactive geometry software package used to help students learn geometry principles. Use of Sketchpad in a secondary education math course at the State University of New York (SUNY) at Oswego has allowed geometry to be taught in a dynamic manner. The paper provides three examples of how Sketchpad is used. Traditional paper folding and ruler and compass constructions are important preliminary steps to teach students, in order to prepare future teachers for classrooms that may not yet have access to dynamic geometry software. Sketchpad provides a measurement option and the ability to do the construction for a single triangle, then click and drag on a vertex to deform the triangle into many different shapes until the important characteristic of the triangle becomes clear. Student reaction to the use of Sketchpad in their geometry course was overwhelmingly positive. Many wanted to purchase copies for their own personal use and for use in their student teaching experience. Both students and instructor learned to use the software to test conjectures and constructions, and back up the theorem/proof process by using Sketchpad to ask "what if" questions. The course came closer to an ideal situation where students construct their own mathematical understandings and where the instructor is less of a depository of mathematical truth but more one of many geometrical investigators. (SWC)

Integrating *Geometer's Sketchpad* into a Geometry Course for Secondary Education Mathematics Majors

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Our Mathematics Department at SUNY Oswego offers a significant number of courses to prospective teachers, at both the elementary and secondary level. Over the years we have attempted to integrate technology into these courses where ever the technology complements the mathematics content of the courses. From the use of the computer language LOGO in courses for elementary education majors, to graphing calculators in the precalculus and calculus sequence, the department has had the opinion that technology should not "stand alone", but be used in a meaningful manner in the mathematics courses. Most recently I have been involved in the use of the interactive geometry software package, Geometer's Sketchpad and would like to report on how the use of Sketchpad has changed the way I teach the course.

In previous semesters the geometry class was taught in a fairly traditional manner. Definitions were given, theorems stated and proved and problems assigned and corrected. We did do some laboratory type activities using MIRAS, a device used to do reflections in a plane, geoboards, paper folding and of course standard ruler and compass constructions. The introduction of Sketchpad allowed us to continue these activities in a dynamic manner. For purposes of illustration I have selected three examples to explain the differences in the course, before and after Sketchpad.

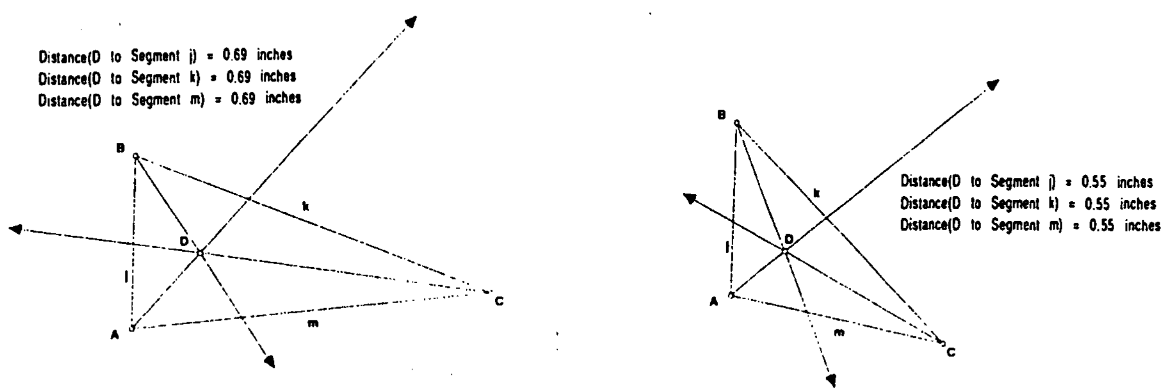
Associated with any triangle are three sets of linesthe angle bisectors, the medians and the altitudes. The angle bisector is the line that bisects one of the three angles in a triangle. The altitude is a line through a vertex and perpendicular to the opposite side, and a median is a line from a vertex of a triangle to the midpoint of the opposite side. We will consider angle bisectors first. Students can use something as simple as paper folding to discover a relationship between the angle bisectors. In triangle ABC side AC is folded over to lie on side AB and the paper is creased. When unfolded the paper crease represents the bisector of angle A. After doing the same for the other two angles the student observes that all three bisectors are concurrent, or contain a common point. Let us state this result as

Proposition 1 The angle bisects of a triangle are concurrent.

After paper folding the instructor can follow with the traditional ruler and compass construction of the angle bisectors of the three angles in a triangle. Again, the bisectors are concurrent at a point in the interior of the triangle. Even after all this, though, the student has done the construction for just a few triangles. Does the result hold in general? Are the angle bisectors of

the angles in a triangle always concurrent or did the student use "special" triangles? The software package Geometer's Sketchpad allows the students to check the conjecture for any triangle they wish to investigate after having constructed the angle bisectors only once. After first constructing an arbitrary triangle and using Sketchpad to bisect the angles of the triangle the student observes that the bisectors are concurrent. So far the technology does nothing that could not be accomplished with paper folding. But Sketchpad allows the student to click and drag on a selected vertex. As the point is dragged, the triangle deforms to different shapes. However, the angle bisectors maintain their status as angle bisectors, and their point of intersection, although changing position, still is a point common to all three angle bisectors. In the diagrams below, point C in triangle ABC was selected and dragged. The bisectors of angles A, B and C remain concurrent at point D and the proposition that angle bisectors are concurrent appears reasonable. It is so reasonable, in fact, that the next struggle is to convince the students of the need for a formal proof.

Figure 1:



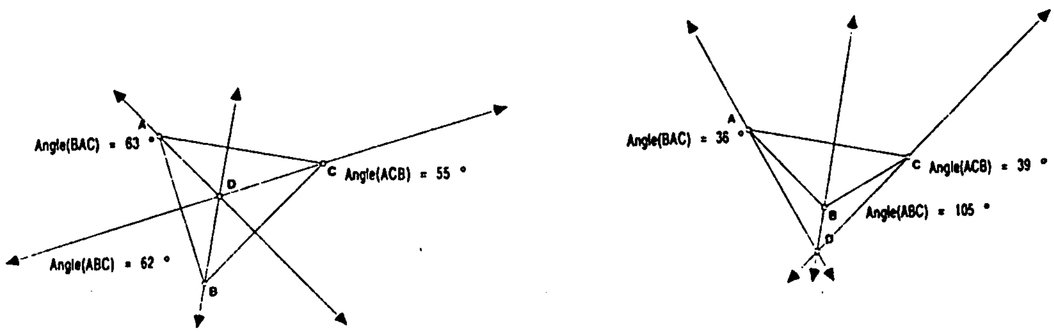
In addition, using the measure option of Sketchpad, the student can measure the distance from point D to each of the three sides of the triangle. As the above figure indicates, the distance is constant in each triangle. This observation allows the student to see a way to devise a formal proof of Proposition 1. The standard proof of this result uses a previous result - the points on an angle bisector are precisely those points that are equidistant from the sides of the angle. Thus, if point D is on the angle bisector of angle A, the distance from D to segment m is equal to the distance from D to segment j. Also, if point D is on the angle bisector of angle B, the the distance from D to segment j is equal to the distance from D to segment k. Thus the distance from point D to segment k equals the distance from D to segment m and D is also on the bisector of angle C. Hence the three angle bisectors are concurrent at point D. The use of Sketchpad has allowed the students to "discover" the proposition, test it to see if it reasonable, and even point the way towards a proof.

A similar result deals with altitudes of a triangle.

Proposition 2 The altitudes of a triangle are concurrent.

The same methodology was employed here. Paper folding gave a rough verification of the result, ruler and compass construction followed and then students were sent to the computer lab to test the result on Sketchpad. I do feel the paper folding and ruler and compass constructions are important preliminaries, in order to prepare our future teachers for classrooms that may not yet have access to dynamic geometry software. The results of a Sketchpad investigation are shown below.

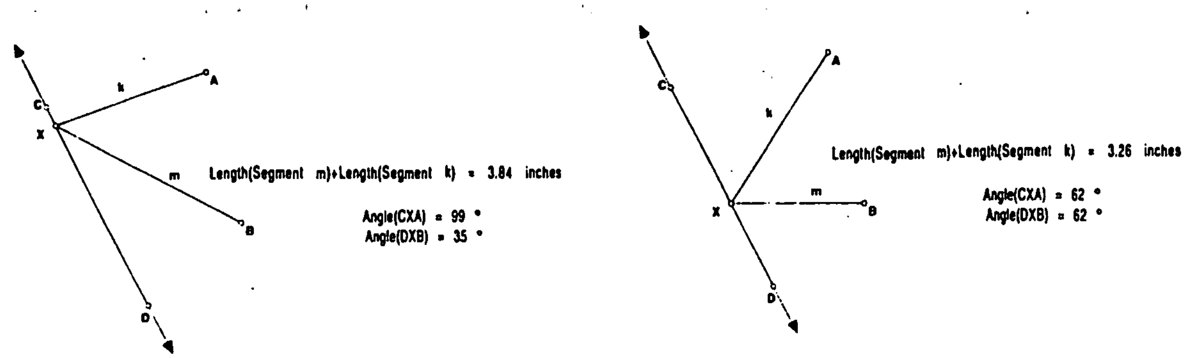
Figure 2



We can carry this example a bit farther than in the investigation of Proposition 1 by asking students to discover when the point common to all three altitudes is not in the interior of the triangle. This is a question that is not possible to consider when doing mechanical ruler and compass construction. With mechanical ruler and compass, the student may get lucky and do the constructions on an initial triangle where the common point is exterior to the triangle, but how will the student know what was special about the triangle? What is needed, and what Sketchpad can provide, is the ability to do the construction for a single triangle, then click and drag on a vertex to deform the triangle into many different shapes until the important characteristic of the triangle becomes clear (answerthe altitudes of a triangle intersect in the interior of the triangle if and only if all angles of the triangle measure less than 90 degrees.) In this example, Sketchpad extends the material that can be investigated.

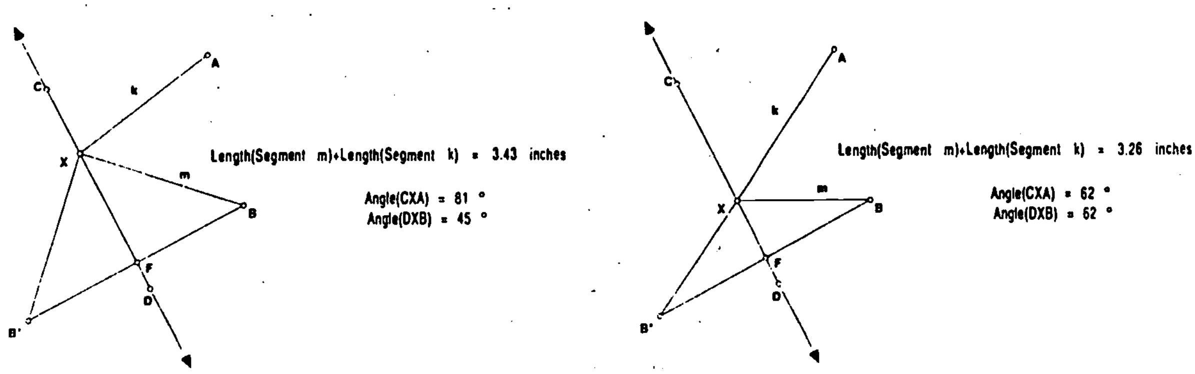
Finally, we see Sketchpad used to investigate a result familiar to pool players. In Figure 3, students were to determine point X on the given line so that the sum of the distances from A to X and from X to B is at a minimum. Actually, in pool the problem is to find the position to bounce ball A off so that it will ricochet and hit ball B. After finding the optimal position for point X students were to print out their sketch, determine the point geometrically and explain why the position they found for point X was optimal.

Figure 3



In Figure 3 two views are shown, with two positions for point X. The view on the right is the optimal position for point X, i.e., the position for which the sum of the distances from point A to point X and from point X to point B are at a minimum. This position is found by using the measure and calculate tool of Sketchpad. As point X is moved up and down on the line the sum of the distances changes. The student simply needs to place point X where this distance sum is at a minimum. Now the problem gets interesting. What is special about the located position? How would you describe this position geometrically? How could you have found the point without using Sketchpad? Many students found these questions difficult. Some noticed that when X is in the optimal position the measure of angle CXA is equal to the measure of angle DXB. But that still didn't give a method for constructing the optimal position or a proof as to why this position would be optimal. The key comes from the material we had done earlier on isometrics of the plane.

Figure 4



Notice that in Figure 4, since B was reflected in line CD to B', line CD is the perpendicular bisector of segment BB'. Thus segment B'F is congruent to segment BF, angle B'FX is congruent to angle BFX and triangle B'FX is congruent to triangle BFX. Since segments BX and B'X are corresponding parts of congruent triangles, they have equal lengths. Minimizing the sum of the distances from A to X and then from X to B now becomes the problem of minimizing the sum of the distances of A to X and from X to B'. If X is in the position that minimizes the sum of the distances, then points A, X and B' are concurrent. If not, points A, X and X' form a triangle. Since the shortest distance between two points is a straight line, the sum of the distances is minimized when A, X and B' are colinear. Thus to construct point X so that the sum of the distances from A to X and from X to B is a minimum first reflect point B through line CD to its image, B'. Then construct a line from B' to A. The point where this line intersects line CD is the position X must occupy to minimize the sum of the distances from A to X and from X to B.

The reaction of the students to the use of Sketchpad in their geometry course was overwhelmingly positive. Many wanted to purchase copies of the software for their own personnel use and for use in their student teaching experience. Both students and instructor learned to turn to the software to test conjectures and constructions. We were able to back the theorem/proof process up a step by using Sketchpad to ask "what if" questions and the course came closer to an ideal situation where students construct their own mathematical understandings. My role as instructor changed from being the depository of mathematical truth to acting as one of many geometrical investigators.

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